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The Decay of Massive Scalar Field in Non-Static Gödel Type Universe with Viscous Fluid and Heat Flow

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Abstract In this study, we have investigated the dynamics of non-static Gödel type rotating universe with massive scalar field, viscous fluid and heat flow in the presence of cosmological constant. For various cosmic matter forms, the behavior of the cosmological constant (Λ), shear (η) and bulk (ξ) viscosity coefficients and other kinematic quantities have studied in the early universe. We have showed the decay of massive scalar field in the non-static rotating Gödel type universe and we have obtained constant scalar field with and without source density. Also, we have investigated the effects of massive scalar field on the matter density and pressure. From solutions of the field equations, we have a cosmological model with non-zero expansion, shear, heat flux and rotation. Also some physical and geometrical aspects of the model discussed.

Keywords Massive scalar field · Gödel universe · Viscous fluid and heat flow

1 Introduction

In cosmology, scalar fields are important for the many researchers. According to Matos et al. [1]; scalar fields as a basic interaction in physics are one of the fundamental predictions of the Kaluza-Klein and the Super-string theories [1]. In the Brans-Dicke theory and its inflationary models, scalar fields are fundamental components. However, they are a good candidate for the dark matter in spiral galaxies [2, 3]. Because they effect so weakly with matter we have never seen one but most of the theories with scalar fields are in good concordance with measurements in weak gravitational fields [1]. According to Singh and Bhamra; scalar meson field which is of zero-mass type and characterizes long-range interactions [4]. Since provide an understanding of the nature of space-time, the investigate of such a field in general relativity has been initiated and the gravitational field associated with neutral elementary particles of zero spin [4]. The idea of scalar fields was introduced by Dirac [5] and

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Kaluza-Klein [6] works and then from Jordan, Brans and Dicke [7], Brans [8] first scalar-tensor theory [9]. According to Fay and Lehner; in cosmology, the recent acceleration of the universe expansion can be explained by a quintessence scalar field and it is also one of the best mechanism to produce inflation at early times. It could even mimic the galactic dark matter [10, 11] responsible for the rotation curves flattening [12]. Mohanty and Sahu [13] examined the zero-mass scalar field in the anisotropic and LRS Bianchi I type universe model when the metric potentials are the functions of the cosmic time t only. Also, they found zero-mass scalar field without source term is compatible and yield feasible solutions, where as zero-mass scalar field with source density model does not survive [14]. In 2002 Mohanty and Mishra [15] discussed that Bianchi I type universe model does not exist in the case of mesonic perfect fluid and meson field with or without the mass parameter (M) in the bimetric theory of gravitation [14]. In 2003 Mohanty et al. [14] studied LRS Bianchi I type universe with attractive massive scalar field in the case of source density model and they have showed that how the dynamical importance of the scalar field and shear change in the course of evolution [14]. Particle physics and high energy astrophysical observations show that the universe is rotating [16–21]. A rotating universe with vanishing expansion and shear appeared with the advent of Gödel's universe [20, 22]. According to Novello and Reboucas, Gödel published the first cosmological model by a solution of the modified Einstein equations in which a cosmological repulsive term (Λg_{ik}) has been added [23]. The congruence of the geodesics of this model has no shear, expansion and no acceleration, but presents a constant rotation of matter relative to the compass of inertia [23]. After this discovery, many attempts were made to construct more general solutions which take the expansion and/or shear into account besides rotation [23, 24]. Now, it is well known that the universe is expanding. Therefore, the study and investigation of non-static rotating world models is important [20]. From this reasons we have selected the Gödel universe. Because it has many interesting feature [25]. This model has natural rotation, even more intriguing is the lack of a global time ordering and existence of closed time-like world lines giving rise to possibility of time travel [26, 27]. Also, this model is geodesically complete, it does not contain horizons or any singularities [28]. The solutions of the various matter distributions with the Gödel universe have been investigated by different authors. Koppar and Patel [29, 30] have presented some non-static generalizations of Gödel solution which rotating universe with viscous fluid and heat flux. Chakraborty and Bandopadhyay [31] have presented a stationary generalization of Gödel's universe with perfect fluid and a scalar field [20]. Non-static Gödel cosmological model with heat flux and perfect fluid have been studied by Yavuz and Baysal [24]. However, usually cosmological models suppose that the matter in the universe can be described by perfect fluid or dust [32]. Nevertheless, there is good reason to believe that—at least in the early stages of universe—viscous effects do play a role [33–37]. For example, the existence of bulk viscosity is equivalent to slow process of restoring equilibrium states [38] and viscosity mechanism can explain the anomalously high entropy per baryon in the present universe [35, 36]. Viscosity plays an important role in explaining many physical features of the homogeneous world models. Homogeneous cosmological models filled with viscous fluid have been fairly studied by Murphy [39], Banerjee and Santos [40] in cosmology. Chimento and Jakubi [41] have studied scalar field cosmologies with viscous fluid. Pradhan et al. [42] have presented some inhomogeneous magnetized viscous fluid cosmological models with varying Λ in Bianchi type universe. The model with shear and bulk viscosity were investigated by Elst [43] and Gavrilov [44]. Furthermore, the matter distribution is not expected to attain thermal equilibrium in the early stages, it is understandable that there would be heat flux in the universe [45]. Several authors [46, 47] have studied the influence of the heat flux in the evolution of the cosmological models [45]. From this point of view, we have investigated the influence of viscous fluid, massive scalar field (in all directions) and heat flow with cosmological constant in the evolution of non-static Gödel type universe. The paper is outlined as follows. In Sect. 2, the Einstein Field Equations are obtained in non-static Gödel model. In Sect. 3, we get some exact solutions for the Einstein Field Equations. In Sect. 4, the results of obtained are discussed.

2 Fundamental Concepts and Field Equations

We consider a non-static Gödel type space-time [22] for rotating universe in the form

$$ds^2 = -dt^2 - \frac{1}{2}H^2 e^{2x} dy^2 - 2He^x dy dt + dx^2 + dz^2 \tag{1}$$

where the metric potential H is a function of cosmic time t only. The Einstein's field equations (in gravitational units $c = 8\pi G = 1$) read as

$$R_{ik} - \frac{1}{2}Rg_{ik} + \Lambda g_{ik} = -T_{ik} \tag{2}$$

where Λ, is the cosmological constant and T_{ik}, is the energy momentum tensor for cosmic matter distribution including massive scalar field, viscous fluid and heat flow and is given

$$\begin{aligned}T_{ik} &= \rho u_i u_k + (p - \xi\theta)h_{ik} - 2\eta\sigma_{ik} + q_i u_k + q_k u_i \\ &\quad + \frac{1}{4\pi}\left[V_{,i} V_{,k} - \frac{1}{2}g_{ik}(V_{,l} V^{,l} - M^2 V^2)\right].\end{aligned} \tag{3}$$

Here p is the isotropic pressure, ρ the fluid density, η, ξ the coefficients of shear and bulk viscosities, respectively and u_i the four vector of the cosmic matter distribution satisfying the relation in co-moving coordinate system.

$$g_{ik}u^i u^k = -1, \qquad u^i = (0, 0, 0, 1), \qquad u_i = (0, -He^x, 0, -1) \tag{4}$$

h_{ik} is projection tensor and given by

$$h_{ik} = g_{ik} + u_i u_k \tag{5}$$

while

$$\sigma_{ik} = \frac{1}{2}\mu_{ik} - \frac{1}{3}\theta h_{ik} \tag{6}$$

is component of shear tensors where,

$$\mu_{ik} = u_{i;k} + u_{k;i} + \dot{u}_i u_k + \dot{u}_k u_i. \tag{7}$$

Here comma and semi-colon denote partial and covariant differentiations, respectively and

$$\theta = u^i_{;i} \tag{8}$$

is the expansion factor. Here q_i is heat conduction vector orthogonal u_i, and

$$q_i q^i > 0; \qquad q_i u^i = 0. \tag{9}$$

However, V is the massive scalar field and we assume that V is a function of the x, y, z, t coordinates $\{V = V(x, y, z, t)\}$. The scalar field V is governed by the Klein-Gordon (KG) equation ($g^{ik}V_{;ik} + M^2V = 0$) as,

$$KG \equiv M^2V + V_{xx} + \frac{2e^{-2x}V_{yy}}{H^2} + V_x + \frac{H_tV_t}{H} - \frac{4e^{-x}V_{yt}}{H} + V_{zz} + V_{tt} = 0 \tag{10}$$

where and after the lower indices x, y, z, t represent the first and second derivatives of x, y, z, t coordinates, respectively. Also, M is related to mass m of zero-spin particle by $M = \frac{2\pi m}{h}$ where h being Planck's constant [48]. Using co-moving coordinates, the field equations (2) for metric (1) with energy momentum tensor (3) can be written as

$$G_{11} \equiv -\frac{H_{tt}}{H} - \frac{1}{2} + \Lambda = \frac{H_t}{H}\left(\xi - \frac{2\eta}{3}\right) - p + \frac{V_y}{2\pi He^x}\left(\frac{V_y}{2He^x} - V_t\right) + \frac{1}{8\pi}(V_z^2 + V_t^2 - V_x^2 - M^2V^2), \tag{11}$$

$$G_{12} \equiv H_te^x - \frac{V_xV_y}{4\pi} + q_1He^x, \tag{12}$$

$$G_{13} \equiv -\frac{V_xV_z}{4\pi}, \tag{13}$$

$$G_{14} \equiv \frac{H_t}{H} - \frac{V_xV_t}{4\pi} + q_1, \tag{14}$$

$$G_{22} \equiv \frac{3}{4} + \frac{\Lambda}{2} = \rho + \frac{p}{2} - \frac{H_t}{H}\left(\frac{\xi}{2} + \frac{2\eta}{3}\right) + \frac{V_y}{4\pi He^x}\left(\frac{3V_y}{2He^x} - V_t\right) + \frac{1}{16\pi}(V_x^2 + V_z^2 + V_t^2 - M^2V^2) - \frac{2q_2}{He^x}, \tag{15}$$

$$G_{23} \equiv q_3He^x - \frac{1}{4}\frac{V_yV_z}{\pi}, \tag{16}$$

$$G_{24} \equiv \frac{1}{2} + \Lambda = \rho - \frac{V_y}{4\pi He^x}\left(V_t - \frac{V_y}{He^x}\right) + \frac{1}{8\pi}(V_x^2 + V_z^2 + V_t^2 - M^2V^2) - \frac{q_2}{He^x}, \tag{17}$$

$$G_{33} \equiv -\frac{H_{tt}}{H} - \frac{1}{2} + \Lambda = \frac{H_t}{H}\left(\xi - \frac{2\eta}{3}\right) - p + \frac{V_y}{2\pi He^x}\left(\frac{V_y}{2He^x} - V_t\right) + \frac{1}{8\pi}(V_x^2 - V_z^2 + V_t^2 - M^2V^2), \tag{18}$$

$$G_{34} \equiv q_3 - \frac{V_zV_t}{4\pi}, \tag{19}$$

$$G_{44} \equiv -\frac{1}{2} - \Lambda = -\rho - \frac{V_y}{2\pi He^x}\left(\frac{V_y}{2He^x} - V_t\right) - \frac{1}{8\pi}(V_x^2 + V_z^2 + 3V_t^2 - M^2V^2). \tag{20}$$

3 Solutions of Einstein Field Equations

The bulk (ξ) and shear (η) viscosities coefficients are both positively definite, i.e.,

$$\eta > 0, \qquad \xi > 0 \tag{21}$$

they may be either constant or function of time [49]. Here we take as constant these coefficients. With (11) and (20) it can be found unknown parameters. Firstly, using (12) and (14) or (16) and (19) we get

$$V_y = V_t He^x. \tag{22}$$

From (11) and (17) we get

$$V_x^2 = V_z^2. \tag{23}$$

From (13) we get two situations about the derivatives of scalar field as follows,

$$V_x = 0 \quad \text{or} \quad V_z = 0. \tag{24}$$

Helping (23) and (24), we obtain independent scalar field from x and z coordinates as follows,

$$V_x = V_z = 0, \quad \text{then } V(x, y, z, t) = V(y, t). \tag{25}$$

In this case, the mesonic scalar field does not exist in the direction of x and z coordinates. Equations (12), (14) and (25) give

$$q_1 = -\frac{H_t}{H}. \tag{26}$$

Helping (19) and (25), q_3 is given by

$$q_3 = 0. \tag{27}$$

From (4) and the condition $q_i u^i = 0$, one obtains $q_4 = 0$. If we compare (25) and (22), we see that (22) must be equal to zero. Because of (25), the left side of (22) depends only y and t coordinates also the right side of (22) depends x, y and t coordinates. From this point of view, (22) is written as

$$V_y = V_t He^x = 0 \quad \text{and} \quad V_y = V_t = 0 \quad (He^x \neq 0). \tag{28}$$

In this case, the mesonic scalar field does not exist in the all directions and we find the constant mesonic scalar field in non-static Gödel space-time $\{V(x, y, z, t) = \beta\}$, here β is a constant. If we substitute (25) and (28) into KG equation (10) we obtain that the mass function of massive scalar field is vanish ($M = 0$) in Gödel type universe and we find zero-mass scalar field in the case of non source density model as follows,

$$KG \equiv M^2 V = 0 \quad \text{and} \quad V = \beta = const. \neq 0, \quad \text{and} \quad M = 0. \tag{29}$$

Using (25), (28) and (29) into (20) we get the fluid density of cosmic matter distribution,

$$\rho = \frac{1}{2} + \Lambda. \tag{30}$$

If we substitute (25), (28), (29) and (30) into (17), q_2 is given by,

$$q_2 = 0. \tag{31}$$

From (15), (25) and (28)–(31) we obtain that the isotropic pressure of the cosmic matter distribution as follows,

$$p = \frac{1}{2} - \Lambda + \frac{H_t}{H}\left(\xi + \frac{4\eta}{3}\right). \tag{32}$$

If we substitute (25), (28), (29) and (32) into (11), the metric potential $H(t)$ is given by,

$$H = b + ae^{2\eta t}. \tag{33}$$

+Here a and b are constants. Also, the non-zero components of the conservation of energy-momentum equations ($T^{ik}_{;\,k} = 0$) are given for this model as the follows,

$$\begin{aligned}
&\frac{H_t}{H}(2\eta + q_1) + q_{1t} = 0,\\
&3H(q_1 + p_t) + 3H_t(p + \rho) - H_{tt}(4\xi + 3\eta) = 0,\\
&3H^2(2p_t + 3q_1 + \rho_t) - (4\eta + 3\xi)(H_t^2 + 2HH_{tt}) + 9HH_t(\rho + p) = 0.
\end{aligned} \tag{34}$$

If we compare the (26) and (32)–(34), we can choose that $b = 0$ in (33) and the metric potential H can be found as

$$H = ae^{2\eta t}. \tag{35}$$

From (26) and (30)–(35), q_1 is given by,

$$q_1 = -2\eta. \tag{36}$$

Using (35) into (32) we find that the isotropic pressure of the cosmic matter distribution as follows,

$$p = \frac{1}{2} - \Lambda + 2\eta\left(\xi + \frac{4\eta}{3}\right). \tag{37}$$

If we investigate the variations of the cosmological constant for this model, we find below results as follows:

(a) *For $p = 0$ Matter Distribution* From (37), the cosmological constant is given by,

$$\Lambda = \frac{1}{2} + 2\eta\left(\xi + \frac{4\eta}{3}\right). \tag{38}$$

(b) *Stiff Matter Distribution* ($p = \rho$) From (30) and (37), the cosmological constant is given by,

$$\Lambda = \eta\left(\xi + \frac{4\eta}{3}\right). \tag{39}$$

(c) *The Radiation Phase* ($p = \frac{\rho}{3}$) From (30) and (37), the cosmological constant is given by,

$$\Lambda = \frac{1}{4} + \frac{3}{2}\eta\left(\xi + \frac{4\eta}{3}\right). \tag{40}$$

(d) *Dark Energy Distribution* ($p = -\rho$) The cosmological constant drops in this case. Also, we find the relation between shear and bulk viscosity coefficients as follows,

$$\xi = -\left(\frac{1}{2\eta} + \frac{4\eta}{3}\right). \tag{41}$$

4 Conclusions and Summary

In this study firstly we have discussed Gödel type non-static, homogeneous, anisotropic and rotating universe with massive scalar field in all directions without source density ($\delta = 0$), viscous fluid and heat flow in the presence of cosmological constant with shear and bulk viscosity coefficients. After the exact solutions of the Einstein's field equations for these matter distributions, we have found that massive scalar field does not exist with heat flow and viscous fluid matter distributions in Gödel type universe and the mass function vanishes ($M = 0$) and massive scalar field turns into constant ($V(x, y, z, t) = \beta$), massless scalar field distribution. Also, we obtained the results of massive scalar field with heat flow and viscous fluid when the massive scalar field $V(x)$, $V(y)$, $V(z)$, $V(t)$ depend on only x, y, z, t coordinates, respectively. So, these results are the same as the results of this paper. However we show that the Λ term plays very important role in the Gödel type space-time. Also, the kinematical quantities in the our model are given as follows, the expansion (θ) is

$$\theta = \frac{\dot{H}}{H} = -q_1 = 2\eta. \tag{42}$$

The shear scalar (σ^2), and the rotation (Ω^2) of the four velocity vector $u_i = [0, -He^x, 0, -1]$ are determined as

$$\sigma^2 = \frac{1}{3}\theta^2 = \frac{4}{3}\eta^2, \tag{43}$$

$$\Omega^2 = \frac{1}{2}. \tag{44}$$

Thus the vorticity remains constant along the whole history of the universe. The acceleration vector ($\dot{u}_i$) and the proper volume ($U^3 = \sqrt{-g}$) are given by,

$$\dot{u}_i = (0, -2\eta a e^{2\eta t + x}, 0, 0), \tag{45}$$

$$U^3 = \frac{\sqrt{2}He^x}{2} = \frac{\sqrt{2}ae^{2\eta t + x}}{2} \tag{46}$$

where g is the determinant of the metric. From (42)–(44), it is easily seen that $\theta, \sigma^2, \Omega^2$ are constants which depend on shear viscosity coefficient. U^3 are functions of x and t for two cases. The proper volume of the universe increases exponentially. This case represents the rotating inflation era in the evolution of the universe. From (45) we have $\dot{u}_i \neq 0$. Therefore, it can be said that the matter filling the universe is not geodetic. From (42) and (43), it is easy to see that

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \simeq 0.577 \tag{47}$$

for our models. The present upper limit of the anisotropy ratio $\frac{\sigma}{\theta}$ is 10^{-3} obtained from indirect arguments concerning the isotropy of the primordial black-body radiation [50]. The ratio $\frac{\sigma}{\theta}$ for our model is considerably greater than its present value. This fact indicates that our solutions represent the early stages of evolution of the universe [20]. Equations (32), (35)–(46) show that the shear coefficient (η) is playing more important role than bulk coefficient (ξ) in this universe model. If we take $\xi = \eta = 0$ in our model, we obtain that the massless and constant scalar field and perfect fluid matter distributions instead of viscous fluid with rotating, non-shear and expanding Gödel type universe model and the heat flow distribution vanishes in this universe model. Also, the various quantities in this new model are given as follows,

$$p=\frac{1}{2}-\Lambda, \tag{48}$$

$$\rho=\frac{1}{2}+\Lambda, \tag{49}$$

$$q_1=q_2=q_3=q_4=0, \tag{50}$$

$$H=a, \qquad \Theta=0, \qquad \sigma^2=0, \qquad \Omega^2=\frac{1}{2}, \tag{51}$$

$$M=0, \qquad V=\beta=const., \qquad U^3=\frac{\sqrt{2}ae^x}{2}, \tag{52}$$

$$\Lambda=\frac{1}{2} \quad (\text{for } p=0); \qquad \Lambda=0 \quad (\text{for } p=\rho); \qquad \Lambda=\frac{1}{4} \quad \left(\text{for } p=\frac{1}{3}\rho\right). \tag{53}$$

If we choose perfect fluid and massive scalar field matter distribution to solve the Einstein field equations, it is easily seen that we find the zero-mass and constant scalar field. These solutions agree with our first model. From these results, we say that without source density ($\delta=0$) in KG equation, the massive scalar field decays to massless, constant scalar field in non-static, rotating Gödel type universe model with or without viscous and heat flow matter distributions.

Until now, helping the KG equation in (10) (i.e. $g^{ik}V_{;ik}+M^2V=0$) we have discussed the scalar meson field without the source density. Now we investigate the massive scalar field with the source density (δ). The Klein Gordon equation corresponding to the scalar field V with the source density is given by

$$KG=g^{ik}V_{;ik}+M^2V=\delta \tag{54}$$

where δ is the source density of the scalar meson field and δ is either a constant or a function of t [14]. Now we investigate the scalar field with the source density in non-static Gödel type universe. From (1)–(9), (11)–(20) and (54), we find the exact solutions of the unknown parameters for massive scalar field with the source density as follows,

$$p=\frac{1}{2}-\Lambda-\frac{M^2\beta^2}{8\pi}+2\eta\left(\xi+\frac{4\eta}{3}\right), \tag{55}$$

$$\rho=\frac{1}{2}+\Lambda+\frac{M^2\beta^2}{8\pi}, \tag{56}$$

$$q_1=-2\eta, \qquad q_2=q_3=q_4=0, \tag{57}$$

$$H=ae^{2\eta t}, \qquad \Theta=2\eta, \qquad \sigma^2=\frac{4\eta^2}{3}, \qquad \Omega^2=\frac{1}{2}, \tag{58}$$

$$M\neq 0, \qquad V=\beta=const., \qquad \delta=M^2\beta=const., \qquad U^3=\frac{\sqrt{2}He^x}{2}. \tag{59}$$

If we investigate the variations of the cosmological constant for this model, we find below results as follows:

(a) *For $p = 0$ Matter Distribution* From (55), the cosmological constant is given by,

$$\Lambda = \frac{1}{2} + 2\eta\left(\xi + \frac{4\eta}{3}\right) - \frac{M^2\beta^2}{8\pi}. \tag{60}$$

(b) *Stiff Matter Distribution* $(p = \rho)$ From (55) and (56), the cosmological constant is given by,

$$\Lambda = \eta\left(\xi + \frac{4\eta}{3}\right) - \frac{M^2\beta^2}{8\pi}. \tag{61}$$

(c) *The Radiation Phase* $(p = \frac{\rho}{3})$ From (55) and (56), the cosmological constant is given by,

$$\Lambda = \frac{1}{4} + \frac{3}{2}\eta\left(\xi + \frac{4\eta}{3}\right) - \frac{M^2\beta^2}{8\pi}. \tag{62}$$

(d) *Dark Energy Distribution* $(p = -\rho)$ The cosmological constant drops in this case. Also, we find the relation between shear and bulk viscosity coefficients as follows,

$$\xi = -\left(\frac{1}{2\eta} + \frac{4\eta}{3}\right). \tag{63}$$

If we substitute $M = 0$ in (55)–(62), we obtain our first results i.e. (30), (32), (36)–(46) and we say that our results agree with each other. From (60)–(62), it is easily seen that the cosmological constant is decreased by the mass function M of the massive scalar field in the case of source density model. From obtained results, the corresponding metric in (1) can now be written with and without source density models in the form

$$ds^2 = -dt^2 - \frac{a^2}{2}e^{2(2\eta t + x)}dy^2 - 2ae^{(2\eta t + x)}dydt + dx^2 + dz^2. \tag{64}$$

Also, from (55) and (56) we found that the massive scalar field exists and the mass function M is not vanishes in this model. Also, in this case of source density model, when cosmic pressure is decreased by the cosmological constant and massive scalar field, the matter density is increased by the cosmological constant and massive scalar field.

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